Properties of Matter

Introduction:

Every Engineer is concerned with the elastic properties of material available to him. he must have a good knowledge of the elastic properties of the materials he proposes to use. This will enable him to predict the behaviour of the materials under the action of deforming forces.

Basic Concepts

(i) Load: The external force acting on a body that produces change in the dimension of the body is called load.

(ii) Deformation: It is the change in dimensions or shape of a body when it is subjected to external forces.

(iii) Restoring force:

When an external force acts on a body to cause deformation, forces of reaction comes into play internally and they to restore the body to its original condition. These internal forces are called restoring forces.

Hooke's law: Robert Hooke an English physicist in the year 1679 had given a relation between stress and strain. This relation is known as Hooke's law.

Statement:

$$\text{Stress} \propto \text{Strain}$$

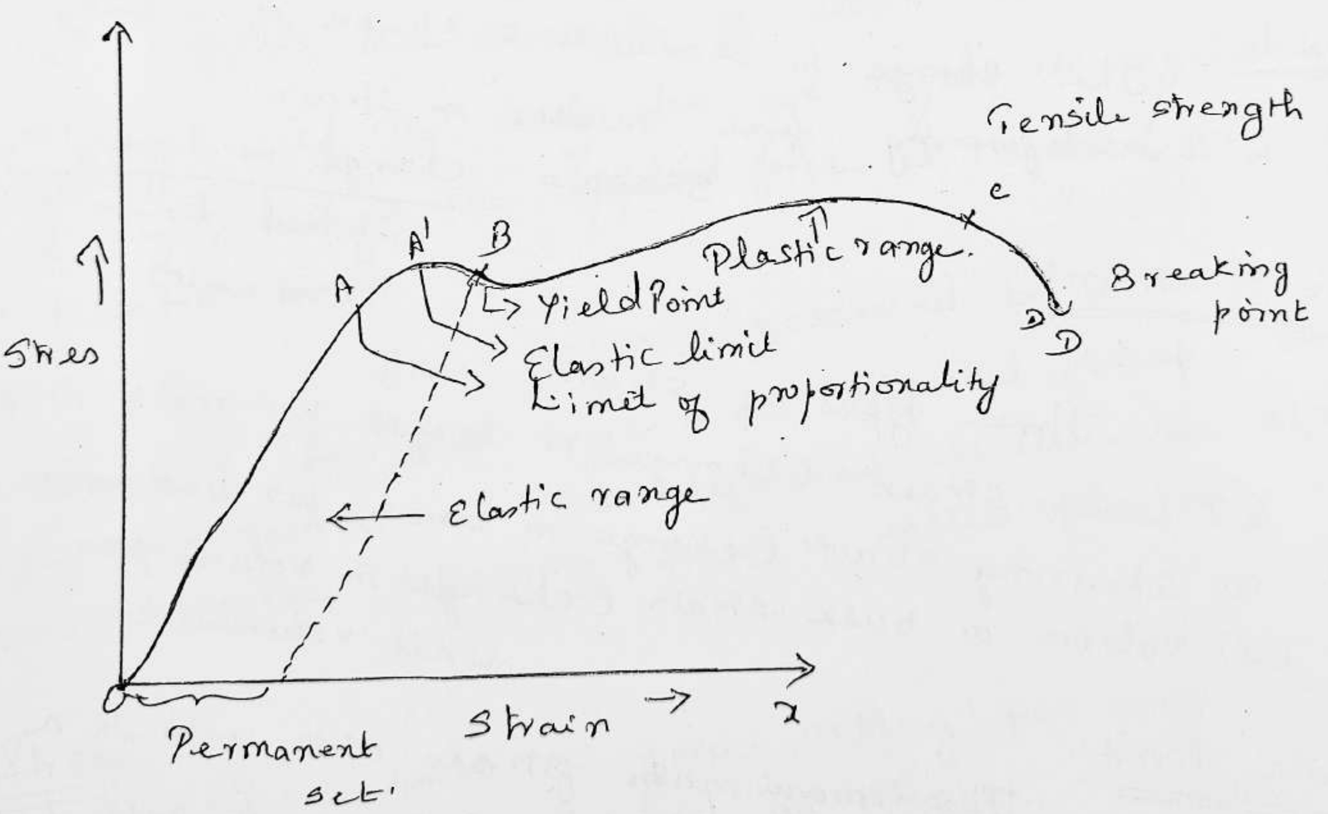
$$\text{stress} = \text{Const} \times \text{strain}$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant } (E).$$

This constant of proportionality is known as co-efficient of elasticity or modulus of elasticity.

'E' is different for different materials.

Stress-strain diagram and its uses



Stress-strain diagram for low carbon-steel wire.

(1) Hooke's law:

The portion 'OA' of the curve is a straight line. In this region stress is directly proportional to strain. This means that upto OA, the material obeys Hooke's law. The wire is perfectly elastic. The point A is called the limit of proportionality.

(2) Elastic limit

The stress is further increased till a point 'A''. The point A' lying near to A denotes the elastic limit. upto this point 'A' the wire regains its original length, if the stress is removed. If the wire is loaded beyond the elastic limit, then it will not restore its original length.

(3) Yield Point:

On further increasing the stress Beyond the elastic limit, the curve bends and a point B is reached.

In this region A' B, a slight increase in stress produces a larger strain in the material. The point B is called Yield Point. The Value of the stress at this point is called yield strength of the material.

(4) Permanent set :-

In the region 'A' & B', if stress is removed, the wire will never return to its original length. The wire is taken a permanent set.

5) Plastic range

Beyond B, the strain in the wire increases rapidly without any increase in the load. This is known as plastic range.

(6) Ultimate tensile strength

If the wire is further loaded, a point C is reached after which the wire begins to neck down. Hence its cross sectional area is no longer uniform.

At this point C, the wire begins to thin down at some point and it finally breaks. At the point C, the stress developed is maximum and it is called ultimate tensile strength or simply tensile strength.

(7) Breaking Point,

The point 'D' is known as the breaking point where the wire breaks down completely. The stress at the point D is called Breaking stress

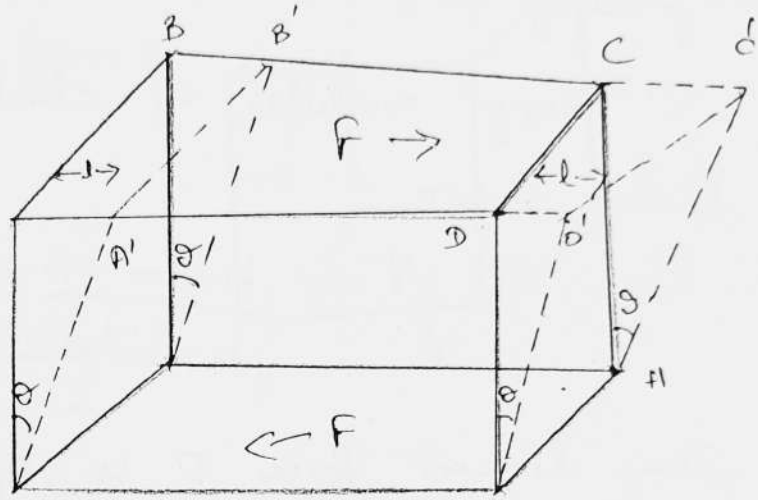
Tensile strength and safety factor

$$\text{Tensile strength} = \frac{\text{Maximum tensile load}}{\text{original cross-sectional area}}$$

$$\text{Safety factor} = \frac{\text{Ultimate tensile stress}}{\text{working stress}}$$

Rigidity modulus of elasticity

$$\text{Rigidity modulus}(n) = \frac{\text{Tangential stress}}{\text{Shearing strain}}.$$



$$\text{Shearing stress} = \frac{\text{Tangential force}}{\text{Area of the face ABCD}}$$

$$= \frac{F}{A}.$$

From the fig $\tan\theta = \frac{AA'}{AF} = \frac{l}{L}$

$\tan\theta = \theta$ (∵) θ is very small.

The angle "θ" is known as the shearing strain or angle of shear.

$$\text{Shearing strain } \theta = \frac{l}{L}$$

$$\text{Rigidity modulus of Elasticity } (n) = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$= \frac{F/A}{l/L} = \frac{FL}{Al}.$$

$$\boxed{n = \frac{FL}{Al}}$$ unit is Nm^{-2}.

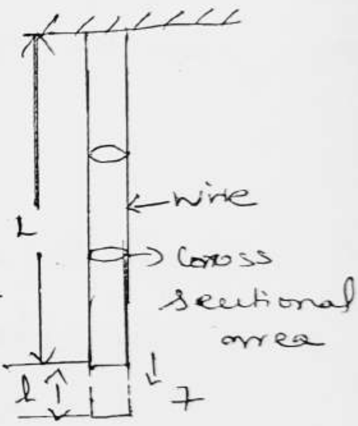
Types of Modulli of Elasticity

They are three types of moduli of elasticity corresponding to three

(i) Youngs modulus of elasticity corresponding to linear strain.

$$Y = \frac{\text{Linear stress}}{\text{Linear strain.}}$$

The linear force F is applied normally to a cross sectional area 'a' of a wire



$$\text{Linear stress} = \frac{\text{Linear force}}{\text{Cross sectional area}} = \frac{F}{a}$$

L is the original length and l is the change in length due to the applied force, then

$$\text{Linear strain} = \frac{\text{change in length}}{\text{Original length}} = \frac{l}{L}$$

$$\text{Youngs modulus of Elasticity} = \frac{\text{Linear stress}}{\text{Linear strain}}$$

$$Y = \frac{F/a}{l/L} = \frac{FL}{al}$$

$$\boxed{Y = \frac{FL}{al}}$$

unit is N/m^2

Nm^{-2}

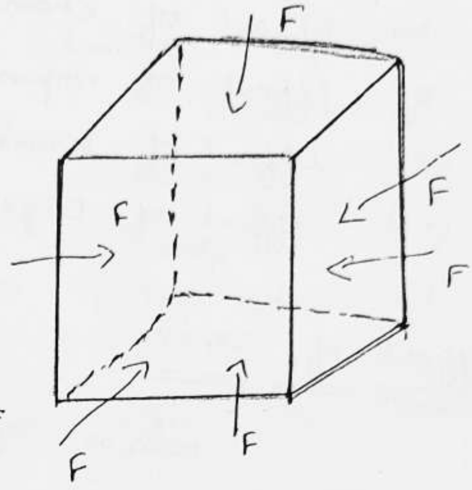
Bulk Modulus of Elasticity (K)

$$\text{Bulk modulus } (K) = \frac{\text{Volume stress}}{\text{Volume strain}}$$

Volume of the body = V

Surface area of each face = A

Subjected to the force



Change in volume = v

$$\text{Volume stress} = \frac{\text{Normal force}}{\text{area}}$$

$$= \frac{F}{A} = P$$

P is the pressure (Force per unit area)

$$\text{Volume strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{v}{V}$$

$$\text{Bulk modulus } (K) = \frac{\text{volume stress}}{\text{Volume strain}} = \frac{P}{v/V}$$

$$\boxed{K = \frac{PV}{v}}$$ unit is Nm^{-2}

Factors Affecting elastic Modulus and Tensile strength

The metal of smaller grains has better elasticity than the same metal of larger grains.

The following factors affect the elastic modulus and tensile strength of the materials. They are

(1) Effect of stress
(2) Effect of change in temperature
(3) Effect of impurities
(4) Effect of hammering, rolling and annealing
(5) Effect of Crystalline nature

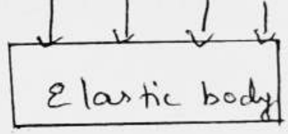
1. Effect of stress

when a small load is applied on the body, elongation occurs immediately on loading and goes back to the original length on removal of the load.

With a higher load, the body continues to stretch, and if the load is removed, a permanent elongation remains.

Stress increases → elasticity of the body decreases.

Application of large Constant stress on the body.



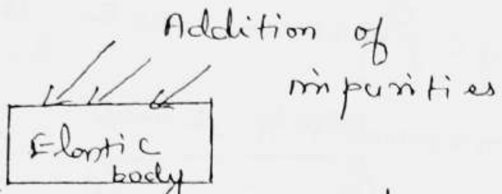
(2) Effect of change in temperature

A change in temperature affects the elastic properties of a material. A rise in temperature usually decreases the elasticity of the material.

Temperature increases, grain size increases, then the distance between atom also increases and so the elastic restoring force decreases. Thus in turn decrease the elasticity.

(3) Effect of Impurities

The elastic property of the material is either increased or decreased due to the addition of impurities. It depends upon the elastic or plastic properties of the impurities added.



The impurities either increase or decrease the elastic properties of the concerned metals.

If the impurity has more elasticity than the material to which it is added, it increases the elasticity, If the impurity is less elastic than the material, it decreases the elasticity.

(4) Effect of hammering, rolling and annealing

A metal with smaller grains has better elasticity than the same metal of larger grains. while being hammered or rolled. crystal grains break into smaller grains resulting in increase of their elastic properties.

Effect of annealing:

while annealing (that is heating and then cooling gradually crystals are uniformly oriented and form larger crystal grains. This results in decrease in their elastic properties.

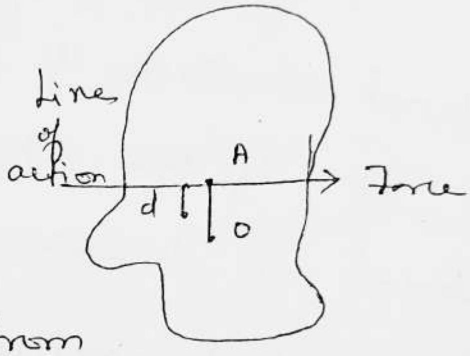
(5) Effect of crystalline nature

for a given metal, the modulus of elasticity is more it is in single crystal form and in polycrystalline state, its modulus of elasticity is comparatively small.

Moment, Couple and Torque

(i) Moment of a force

The moment of a force about a point is defined as the product of magnitude of the force and the perpendicular distance from the point to line ~~li~~ of action of force.



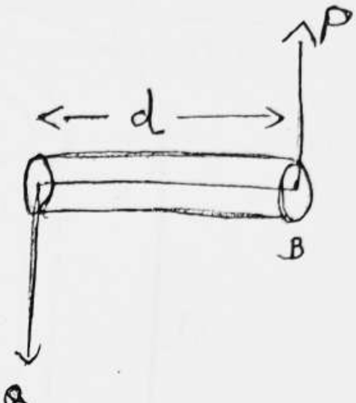
Let 'F' be the force acting on a body, at A as shown in figure.

Then, the moment of force 'F' about 'O' is

$M = F \times d$, where 'd' is the perpendicular distance from the point 'O' to the line of action of force 'F'.

(ii) Couple:

A couple constitutes a pair of two equal and opposite forces acting on a body, in such a way that the lines of action of the two forces are not in the same straight line.



Let 'P' and 'Q' be the two equal and opposite forces acting on the body AB as shown in figure.

These two forces form a couple and the moment of the couple about A is M_A and about B is M_B, then we can write

$$\boxed{\text{Couple} = M_A = M_B = P \times d = Q \times d}$$

Torque: (τ) of a force with respect to a fixed point is defined as the product of the force 'F' and the perpendicular distance (d) of the fixed point from the line of action of the force.

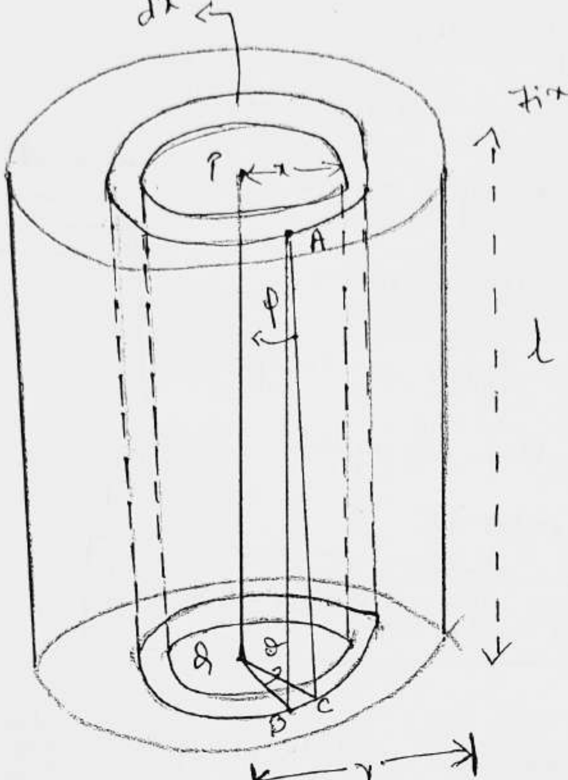
$$\text{Torque } \tau = F \times d.$$

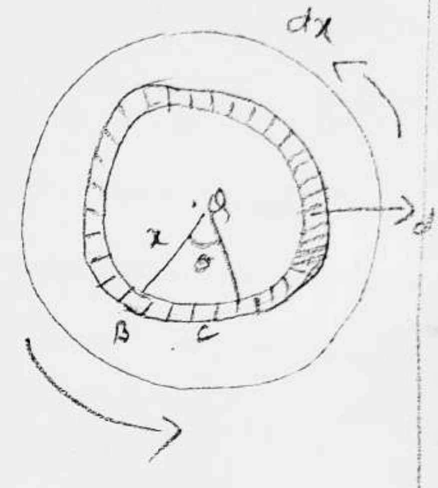
Application of elasticity to torsion of wires or cylinders or shafts

The concepts of elasticity can be applied to the torsion of wires or cylinders and torsion pendulum.

Twisting couple on a wire

Consider a cylindrical wire of length 'l' and radius r fixed at one end. (fig).





It is twisted through an angle θ by applying couple to its lower end. Now the wire is said to be in under Torsion.

Internal restoring couple is equal and opposite to the external twisting couple.

The cylinder consists of a large number of thin hollow coaxial cylinders.

Consider one such cylinder of radius 'x' and thickness 'dx'.

AB is a line parallel to PQ

AB is shifted to AC through an angle

$BAC = \varphi$.

Shearing strain (or) Angle of shear $= \varphi$

Angle of twist at the free end $= \theta$

From the figure:

$$BC = x\theta = l\phi$$

$$\phi = \frac{x\theta}{l}$$

Rigidity modulus $n = \frac{\text{Shearing stress}}{\text{shearing strain}}$

Shearing stress = $n \times$ Shearing strain

$n\phi$

$$\boxed{\text{Stress} \Rightarrow \frac{n x \theta}{l}} \quad -(1)$$

Shearing stress = $\frac{\text{shearing force}}{\text{Area over which the force acts}}$

Shearing force = Shearing stress × Area.

Area over which the force act = $\pi(x+dx)^2 - \pi x^2$

$\pi(x^2 + 2x dx + dx^2) - \pi x^2$

$\pi x^2 + 2\pi x dx + \pi dx^2 - \cancel{\pi x^2}$

dx^2 is neglected

$\Rightarrow 2\pi x dx$

$F \Rightarrow n$ ~~$2\pi x dx$~~ $\frac{n x \theta}{l} 2\pi x dx$

$$\boxed{F \Rightarrow \frac{2\pi n \theta}{l} x^2 dx} \quad -(2)$$

Moment of this force about the axis PQ of the cylinder

= Force × ⊥r distance

$= \frac{2\pi n \theta x^2 dx \cdot x}{l}$

$$\boxed{= \frac{2\pi n \theta}{l} \cdot x^3 dx} \quad -(3)$$

The moment of the force for the entire cylinder of radius r.

twisting couple $c = \int_0^r \frac{2\pi n\theta}{l} x^3 dx$

$$= \frac{2\pi n\theta}{l} \int_0^r x^3 dx$$

$$= \frac{2\pi n\theta}{l} \left[\frac{x^4}{4}\right]_0^r$$

$$\Rightarrow \frac{2\pi n\theta}{l}\left[\frac{r^4}{4} - 0\right] = \frac{2\pi n\theta r^4}{4l}$$

$$\boxed{c = \frac{n\pi\theta r^4}{2l}} \quad (4)$$

If $\theta = 1$ radian ∴

Twisting couple per unit twist

$$\boxed{c = \frac{n\pi r^4}{2l}} \quad (5)$$

Hollow cylinder :-

For a hollow cylinder of the same length l and of inner radius r_1 and outer radius r_2

Twisting couple of the cylinder $C = \int_{r_1}^{r_2} \left(\frac{2\pi n\theta}{l}\right) x^2 dx$

$$= \frac{\pi n\theta}{2l}\left(r_2^4 - r_1^4\right)$$

Twisting couple per unit twist ($\theta = 1$ rad)

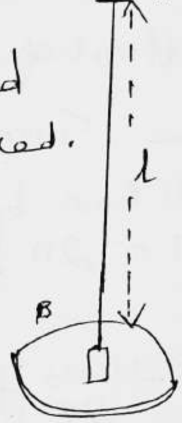
$$C = \frac{n\pi}{2l}\left(r_2^4 - r_1^4\right) \quad (6)$$

Torsion Pendulum - Theory and Experiment

A circular metallic disc suspended using a thin wire that executes torsional oscillation is called torsional pendulum.

* Torsional pendulum executes torsional oscillations, whereas a simple pendulum executes linear oscillations.

Explanation:



A torsional pendulum consists of a metal wire suspended vertically with the upper end fixed. The lower end of the wire is connected to the centre of a heavy circular disc.

The wire is twisted through an angle θ.

The restoring couple $= C\theta$ — (1)

where 'C' is the couple per unit twist. If the disc is released, it oscillates with angular velocity $\frac{d\theta}{dt}$. These oscillations are known as torsional oscillations.

$\frac{d^2\theta}{dt^2} \rightarrow$ angular acceleration.

$I \rightarrow$ moment of Inertia.

Applied couple $= I \frac{d^2\theta}{dt^2}$ — (2)

In Equilibrium applied couple = restoring couple

$$I \frac{d^2\theta}{dt^2} = C\theta$$

$$\frac{d^2\theta}{dt^2} = \frac{C}{I}\theta \quad -(3)$$

angular acceleration $\left(\frac{d^2\theta}{dt^2}\right)$ is always proportional to angular displacement 'θ' and is always directed towards the mean position.

Hence Motion of the disc is simple harmonic motion. The Time period of the oscillation is given by

$$T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$T = 2\pi \sqrt{\frac{\theta}{\frac{C}{I}\theta}}$$

$$\boxed{T = 2\pi \sqrt{I/C}} \quad -(4)$$

Uses of Torsional Pendulum

(1) Rigidity modulus of the wire
(2) Moment of inertia of the disc
(3) Moment of inertia of an irregular body.

Determination of Rigidity modulus of the wire:

The rigidity modulus of the wire is determined by the following equation

$$T = 2\pi \sqrt{I/C} \quad -(1)$$

Experiment :- A circular disc is suspended by a thin wire, whose rigidity modulus is to be determined. The top end of the wire is fixed firmly in a vertical support.

The time taken for 20 oscillations is noted. The experiment is repeated and the mean time period (T) of oscillation is determined.

The length 'l' of the wire is measured. The readings for five or six different lengths of wire are measured.

The disc is removed and its mass and diameter are measured.

The time period of oscillation is

$$T = 2\pi\sqrt{I/C} \quad \text{—} (2)$$

Squaring on both sides, we have

$$T^2 = 2^2\pi^2\left(\sqrt{I/C}\right)^2 \quad \text{—} (3)$$

$$T^2 = \frac{4\pi^2 I}{C} \quad \text{—} (4)$$

Substituting couple per unit twist $C = \frac{\pi n r^4}{2l}$

in eq (4)

$$T^2 = \frac{4\pi^2 I}{\frac{n\pi r^4}{2l}} \Rightarrow \frac{2l \times 4\pi^2 I}{n\pi r^4}$$

$$\boxed{T^2 \Rightarrow \frac{8\pi l I}{r^4 n}}$$

$$\boxed{n \Rightarrow \frac{8\pi I l}{T^2 r^4}}$$

The rigidity modulus of the material of the wire

$$n = \frac{8\pi I l}{T^2 r^4}$$

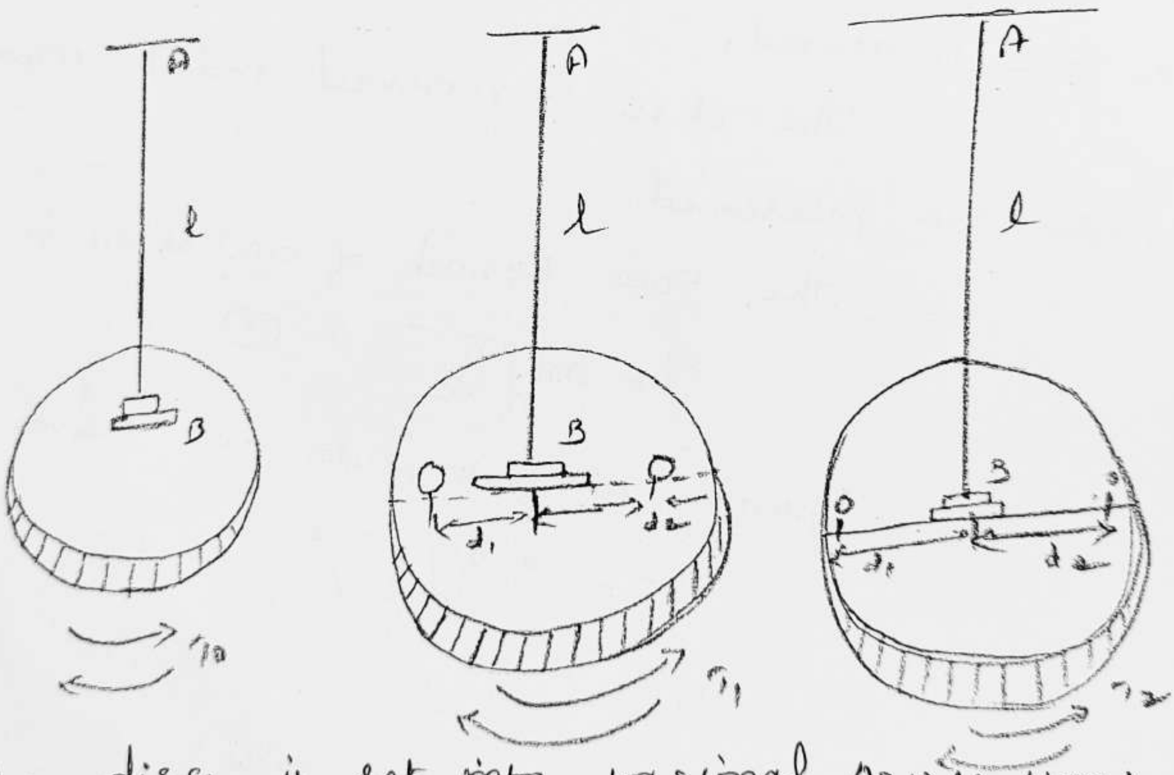
I = moment of inertia of circular disc = $\frac{MR^2}{2}$

$M \rightarrow$ Mass of the circular disc

$R \rightarrow$ Radius of the disc //

Rigidity modulus by Torsion Pendulum (Dynamic Torsion method)

The experiement Consists of three parts.



First The disc is set into torsional Oscillations without any cylindrical masses on the disc.

$$T_0 = 2\pi\sqrt{I_0/C}$$

$I_0 \rightarrow$ moment of inertia of the disc about the axis of the wire

$$T_0^2 = 4\pi^2 \frac{I_0}{C} \quad \text{— (1)}$$

(ii) Two equal cylindrical masses (each mass on equal to 200 gm) are placed symmetrically along a diameter of the disc at equal distance d_1 on the two sides of the centre of the disc.

Mean Time period of oscillation T_1 is found

$$T_1 = 2\pi\sqrt{I_1/C}$$

$$T_1^2 = \frac{4\pi^2 I_1}{C} \quad \text{— (2)}$$

Then by the parallel axis theorem, the moment of inertia of the whole system is given by

$$I_1 = I_0 + 2i + 2md_1^2 \quad \text{— (3)}$$

Substitute the value of I_1 in eq (2)

$$T_1^2 = \frac{4\pi^2}{C}(I_0 + 2i + 2md_1^2)$$

Now two cylindrical masses are placed symmetrically at equal distances d_2

Time period of oscillation T_2 is found

$$T_2 = 2\pi\sqrt{\frac{I_2}{C}}$$

$$T_2^2 = \frac{4\pi^2 I_2}{C} \quad \text{—}$$

$$T_2^2 \Rightarrow 4\pi^2 \frac{(I_0 + 2i + 2md_2^2)}{C} \quad \text{— (5)}$$

$$I_2 - I_1 = I_0 + 2i + 2md_2^2 - I_0 - 2i - 2md_1^2$$

$$\Rightarrow 2m(d_2^2 - d_1^2).$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{C}\, 2m(d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{c}(I_2 - I_1) \quad \text{—} \quad (6)$$

Dividing eq (1) by eq (6)

$$\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{I_2 - I_1}$$

$$\frac{T_0^2}{T_2^2 - T_1^2} = \frac{I_0}{2m(d_2^2 - d_1^2)}$$

$$\boxed{I_0 = \frac{2m(d_2^2 - d_1^2)\, T_0^2}{T_2^2 - T_1^2}}$$

Thus the moment of inertia of the disc about the axis of rotation is calculated.

<u>Calculation of rigidity modulus of the wire</u>:

restoring couple per unit twist

$$c = \frac{\pi n r^4}{2l}$$

$$T_2^2 - T_1^2 = \frac{4\pi^2}{\frac{\pi n r^4}{2l}} 2m(d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 \Rightarrow \frac{8\pi^2 l}{\pi n r^4} 2m(d_2^2 - d_1^2)$$

$$T_2^2 - T_1^2 \Rightarrow \frac{16\pi m l(d_2^2 - d_1^2)}{n r^4}$$

$$\boxed{n \Rightarrow \frac{16\pi m l\,(d_2^2 - d_1^2)}{(T_2^2 - T_1^2) r^4} \; Nm^{-2}}$$

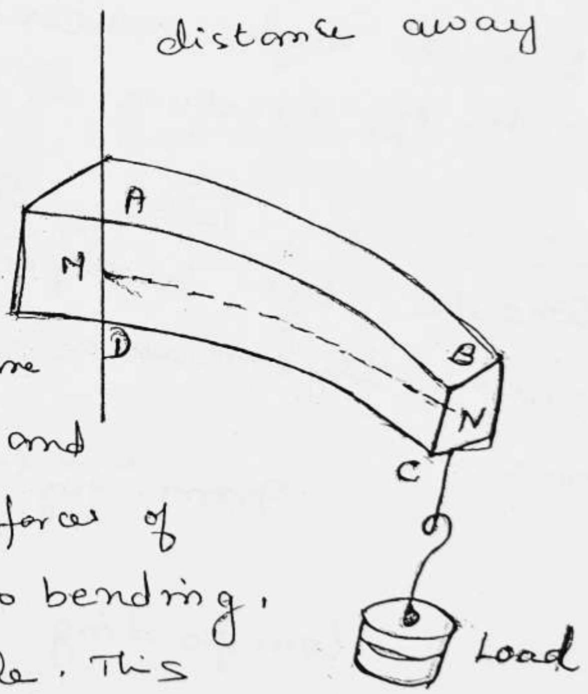
Rigidity modulus of the wire is determined.

Bending of Beam

Taking a longitudinal section ABCD of the bent beam, the layers in the upper half are elongated while those in the lower half are compressed

In the middle, there is a layer (MN) which is not elongated or compressed due to bending of the beam. This layer is called 'neutral surface'. and the line (MN) at which the neutral layer intersects the plane of bending is called 'neutral axis'.

It is found that the length of the layers increases or decreases in proportion to its distance away from the neutral axis MN.

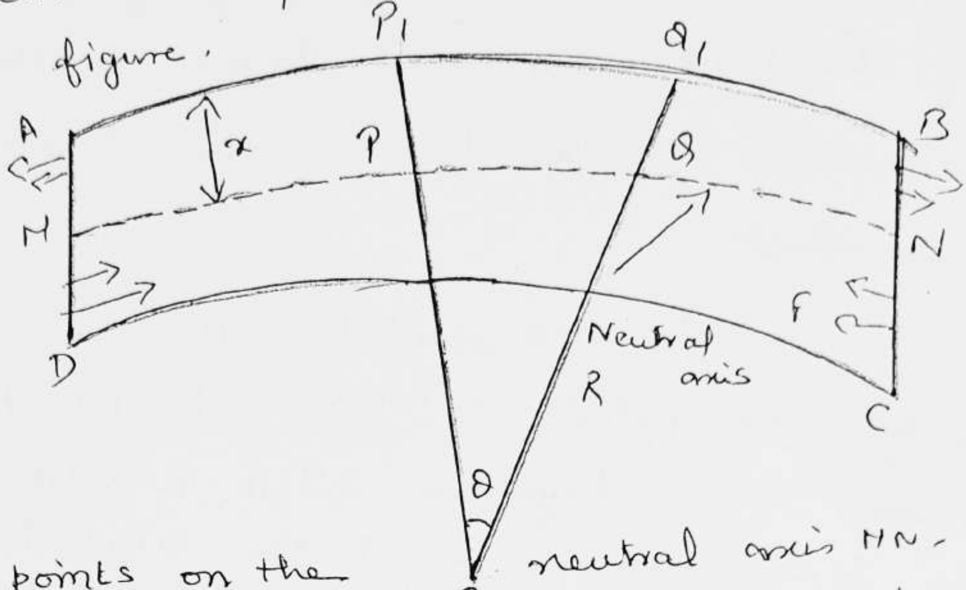


The layers below MN are compressed and those above MN are elongated. There are a pairs of layers one above MN and one below MN experiencing same forces of elongation and compression due to bending. Each pair of layers forms a couple. This couple is known as internal couple.

"The resultant of the moments of all these internal couples are called "internal bending moment".

Bending Moment of a Beam :-

Consider a portion ABCD of a bent beam as shown in figure.



P and Q are two points on the neutral axis MN. R is the radius of curvature of the neutral axis and θ is the angle subtended by the bent beam at its centre of curvature O.

$$\angle POQ = \theta$$

consider two corresponding points P_1 and Q_1 on a parallel layer at a distance x from the neutral axis.

From fig $PQ = R\theta$ — (1)

Corresponding length on the parallel layer

$$P_1Q_1 = (R+x)\theta$$

Increase in length of P_1Q_1

$$= P_1Q_1 - PQ$$

$$(R+x)\theta - R\theta$$

$$R\theta + x\theta - R\theta$$

$$= x\theta \quad \text{— (2)}$$

Linear strain produced = $\frac{\text{Increase in length}}{\text{original length}}$

$$\Rightarrow \frac{x\theta}{R\theta} = \frac{x}{R} \quad — (3)$$

Y is Young's modulus of the material, then

$$Y = \frac{\text{linear stress}}{\text{linear strain}}$$

$$\text{Linear stress} = Y \times \text{Linear strain}$$

$$= \frac{Yx}{R} \quad — (4)$$

δA is the area of cross section of the layer,

then

Force acting on the area

$$\delta A = \text{stress} \times \text{area}$$

$$= \frac{Yx}{R}\delta A \quad — (5) \qquad \text{stress} = \frac{\text{force}}{\text{area}}$$

Moment of this force about the neutral axis MN

Force × ⊥r distance

$$\Rightarrow \frac{Yx\,\delta A}{R} \times x$$

$$= \frac{\Sigma\, Y \delta A\, x^2}{R}$$

$$= \Sigma \frac{Y I}{R}$$

$\Sigma \delta A \cdot x^2 = I$ is called geometrical moment of inertia of the cross section of the beam.

The sum of moments of force acting on all the layers is the internal bending moment and which comes into play due to elasticity.

Internal bending moment of the beam $= \frac{YI}{R}$

$$\boxed{y = \frac{YI}{R}}$$

Rectangular beam

$$I = \frac{bd^3}{12}$$

Circular cross section : $I = \frac{\pi r^4}{4}$.

<u>Stress due to bending in Beams</u>

$$\text{Strain} = \frac{x}{R}$$

$$\varepsilon = \sigma/Y = \frac{\text{Stress}}{\text{Youngs modulus}}$$

$$\frac{\sigma}{Y} = \frac{x}{R}$$

(or)

$$\boxed{\sigma = \left(\frac{x}{R}\right)Y}$$

$\left(\frac{x}{R}\right)$ is Constant.

<u>Cantilever - Theory and Experiment</u>

<u>Definition:</u>

It is a beam fixed horizontally at one end and loaded at the other end.

$$W(l-x) = \frac{YI}{R} \quad \text{—} ①$$

Q is another point at a distance dx from P

$$PQ = dx$$

O is the centre of curvature of the arc PQ.

$$PO = R \text{ and } \angle POQ = d\theta$$

$$\cancel{dx} \; dx = R\,d\theta \quad \text{—} ②.$$

Vertical depression $cd = dy = (l-x)d\theta$ — ③

from eq ② & ③ we have

$$\frac{dx}{dy} = \frac{R\,d\theta}{(l-x)\,d\theta} = \frac{R}{l-x}$$

$$R \Rightarrow \frac{(l-x)\,dx}{dy} \quad \text{—} ④$$

Sub R in eq ①

$$W(l-x) = \frac{YI\,dy}{(l-x)\,dx} \quad \text{—} ⑤$$

$$\frac{W(l-x)^2\,dx}{YI} = dy$$

$$\boxed{\frac{W\,(l-x)^2\,dx}{YI} = dy} \quad \text{—} ⑥$$

Total depression $(y = BB')$ at the free end is

$$\int dy = \int_0^l \frac{W}{YI}(l-x)^2\,dx \quad \text{—} ⑦$$

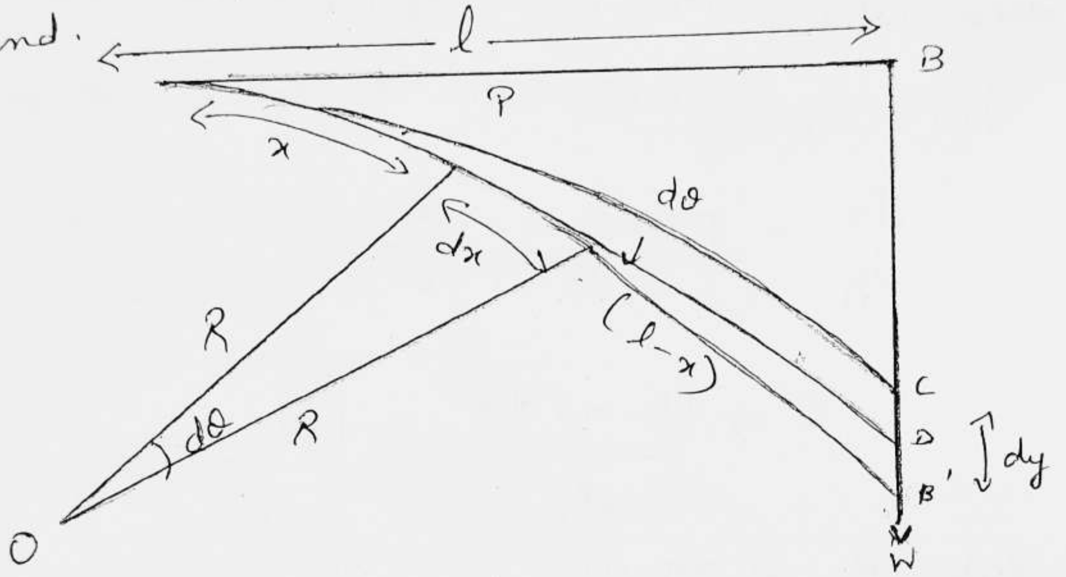
$$y = \frac{W}{YI}\int_0^l (l-x)^2\,dx$$

$$y = \frac{W}{YI}\int_0^l (l^2 + x^2 - 2lx)\,dx$$

Expression for depression produced in the cantilever

Consider a cantilever of length 'l' fixed at the end A and loaded at the free end 'B' by a weight 'W'. The end B is depressed to B'. AB is the neutral axis.

BB' represents the vertical depression at the free end.



Consider the section of the cantilever P at a distance 'x' from the fixed end A. It is at a distance $(l-x)$ from the loaded end 'B'.

$$\text{External bending moment} = W \times PB'$$
$$= W(l-x)$$

$$\text{Internal bending moment} = \frac{YI}{R}$$

Y - Young's modulus of the cantilever
I → Moment of inertia of its cross section
R → Radius of the curvature of the neutral axis at P.

In equilibrium Position
External bending = Internal bending

$$y = \frac{W}{YI}\left[l^2x + \frac{x^3}{3} - \frac{2lx^2}{2}\right]_0^l$$

$$y = \frac{W}{YI}\left(\cancel{l^3} + \frac{l^3}{3} - \cancel{l^3}\right]$$

$$y = \frac{W}{YI} \times \frac{l^3}{3}$$

$$\boxed{y = \frac{Wl^3}{3YI}} \quad \text{--- (8)}$$

Determination of Young's modulus of the cantilever

Depression produced

$$y = \frac{Wl^3}{3YI}$$

$$Y = \frac{Wl^3}{3Iy} \quad \text{--- (9)}$$

For a beam of rectangular cross section

$I = \frac{bd^3}{12}$ b is breadth of the beam
'd' is thickness of the beam

$W = Mg$, M is the mass suspended at the free end and g is acceleration due to gravity

Substituting for W & I in eq (9) we have

$$Y = \frac{Mgl^3}{\frac{3bd^3}{12}y}$$

$$Y \Rightarrow \frac{Mgl^3}{\frac{bd^3 y}{4}} \Rightarrow \frac{4Mgl^3}{bd^3 y}.$$

$$\boxed{Y = \frac{4Mgl^3}{bd^3 y}} \ N/m^2$$ Y is determined.

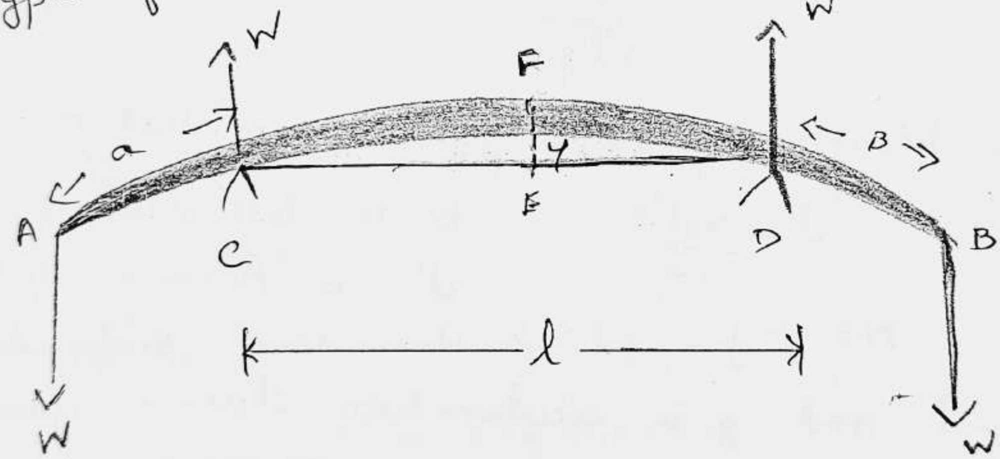
Experimental determination of Young's Modulus

Cantilever

Load	Microscope readings for depression (y)			Mean depression 'y' for a load of M 'kg'.
	Load increa	Loading decreasing	Mean	

Uniform Bending (Theory and Experiment)

Definition:- If the beam is loaded uniformly on its both ends, bending of the beam forms an arc of a circle. The elevation is produced in the beam. This type of bending is known as uniform bending.



Theory: Consider a beam AB arranged horizontally on two knife edges C and D symmetrically so that $AC = BD = a$. The beam is loaded with equal weight 'W' at each ends A and B.

External bending moment on the part AF of the beam about the F is

$$W \times AF - W \times CF = W(AF - CF)$$

$$W \times AC = W \times a = Wa \quad \text{—} (1)$$

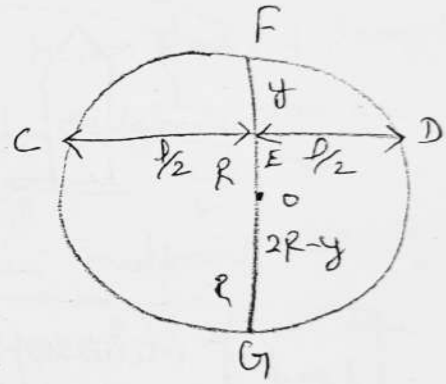
Internal bending moment $= \frac{YI}{R}$ — (2)

External = Internal

$$Wa = \frac{YI}{R} \quad \text{— (3)}$$

for a given value of W, the value of a, Y and I are constant, R is constant so that the beam bends uniformly into an arc of a circle of radius R as shown

CD = l, y is the elevation of the midpoint E of the beam so that y = EF.



From property of a circle

$$EF \times EG = CE \times ED \quad \text{— (4)}$$

$$EF(2R - EF) = (CE)^2$$

$$y(2R - y) = (l/2)^2$$

$$2Ry - y^2 = \frac{l^2}{4}$$ y^2 is negligible

$$2Ry = \frac{l^2}{4}$$

$$y = \frac{l^2}{8R}$$

$$\frac{8y}{l^2} = \frac{1}{R} \quad \text{— (5)}$$

Sub eq (5) in (3)

$$\cancel{Wa = \frac{YI}{8y/l^2}} \Rightarrow$$

$$Wa = \frac{YI\,8y}{l^2}$$

$$y = \frac{Wal^2}{8IY}$$

$$\boxed{Y = \frac{Wal^2}{8IY}}$$

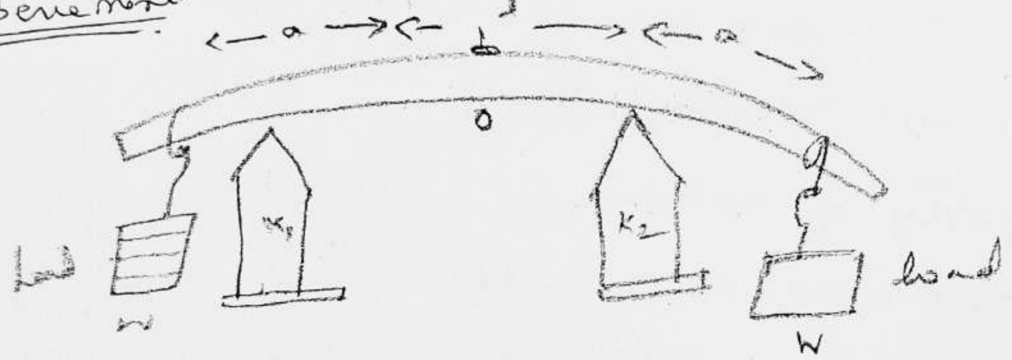
If the beam is of rectangular cross section

then $I = \frac{bd^3}{12}$

$$Y = \frac{Mgal^2}{8\frac{bd^3}{12}y}$$

$$\boxed{Y = \frac{3}{2}\frac{Mgal^2}{bd^3y}.}$$

Experiment.

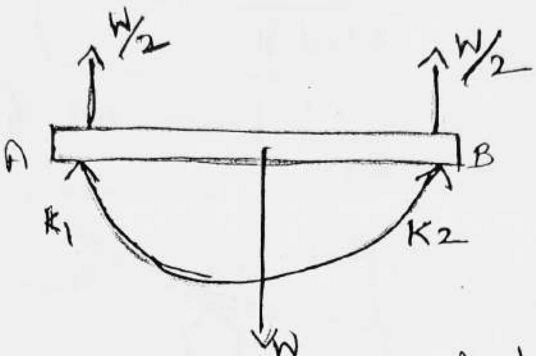


Load gm	Microscope reading		mean cm	Mean elevation for the load of mass m kg
	loading cm	unloading cm.		

$$\boxed{Y = \frac{3}{2}\frac{Mgal^2}{bd^3y} \quad N/m^2}$$

Non-Uniform Bending

If the beam is loaded at its midpoint, the depression produced does not form an arc of a circle. This type of bending is called non-uniform bending.



Consider a uniform cross sectional beam (AB) of length 'l' arranged horizontally on two knife edges K_1 and K_2 near the ends A and B, as shown in fig.

A weight 'W' is applied at the midpoint 'O' of the beam. The reaction force is equal to $\frac{W}{2}$ in the upward direction. y is the depression at the midpoint 'O'.

The bent beam is equivalent to two inverted cantilevers fixed at O each of length $(l/2)$. And each loaded at K_1 & K_2 with ~~respect to~~ weight $W/2$.

In the case of cantilever of length l, and load ~~and entropy~~ the depression is

$$y = \frac{\left(\frac{W}{2}\right)\left(l/2\right)^3}{3Iy} = \frac{\frac{W}{2}\,\frac{l^3}{2^3}}{3Iy} = \frac{\frac{Wl^3}{16}}{3Iy}$$

$$y = \frac{Wl^3}{48Iy}.$$

$$\boxed{\cancel{y = \frac{l}{8}\ldots} \quad \cancel{y = \frac{Wl^3}{48Iy}}}$$

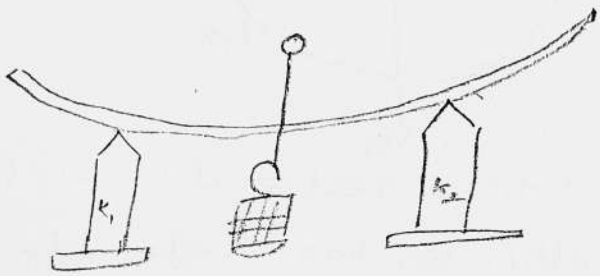
$$Y = \frac{Wl^3}{48Iy}$$

$$W = mg, \quad I = \frac{bd^3}{12}$$

$$Y = \frac{Mgl^3}{48\left(\frac{bd^3}{12}\right)y} \Rightarrow \frac{Mgl^3 \times 12}{48bd^3y}$$

$$\boxed{Y = \frac{Mgl^3}{4bd^3y} \ N/m^2}$$

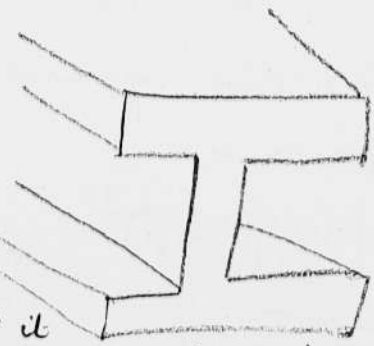
Experiment :- A given beam AB of rectangular



cross section is arranged horizontally on two knife edges K_1 and K_2 near the ends A & B.

Load in kg gm	Microscope reading for depression			mean depression of load kg cm.
	loading cm	unloading cm	mean cm	

I - Shape Girder



The girder with lower and upper section broadened and the middle section tapered. So that it can withstand heavy loads over it is called I shape girders. The cross section of girder look like letter I, & is named as I - shape girder.